# Summer Work Booklet

## Get ready for
## A-Level Maths

This booklet contains examples and questions on topics from your GCSE that you need to be good at to succeed at A-level Maths. Work through the examples then try the questions.

Name ___________________________________

# Rearranging equations

## Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

## Examples

**Example 1** Make $t$ the subject of the formula $v = u + at$.

| | |
|---|---|
| $v = u + at$ | **1** Get the terms containing $t$ on one side and everything else on the other side. |
| $v - u = at$ | |
| $t = \frac{v-u}{a}$ | **2** Divide throughout by $a$. |

**Example 2** Make $t$ the subject of the formula $r = 2t - \pi t$.

| | |
|---|---|
| $r = 2t - \pi t$ | **1** All the terms containing $t$ are already on one side and everything else is on the other side. |
| $r = t(2 - \pi)$ | **2** Factorise as $t$ is a common factor. |
| $t = \frac{r}{2-\pi}$ | **3** Divide throughout by $2 - \pi$. |

**Example 3** Make $t$ the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

| | |
|---|---|
| $\frac{t+r}{5} = \frac{3t}{2}$ | **1** Remove the fractions first by multiplying throughout by 10. |
| $2t + 2r = 15t$ | **2** Get the terms containing $t$ on one side and everything else on the other side and simplify. |
| $2r = 13t$ | |
| $t = \frac{2r}{13}$ | **3** Divide throughout by 13. |

**Example 4** Make $t$ the subject of the formula $r = \frac{3t+5}{t-1}$.

| | |
|---|---|
| $r = \frac{3t+5}{t-1}$ | **1** Remove the fraction first by multiplying throughout by $t - 1$. |
| $r(t - 1) = 3t + 5$ | **2** Expand the brackets. |
| $rt - r = 3t + 5$ | **3** Get the terms containing $t$ on one side and everything else on the other side. |
| $rt - 3t = 5 + r$ | |
| $t(r - 3) = 5 + r$ | **4** Factorise the LHS as $t$ is a common factor. |
| $t = \frac{5+r}{r-3}$ | **5** Divide throughout by $r - 3$. |

## Practice

Change the subject of each formula to the letter given in the brackets.

**1** $C = \pi d$ [$d$]

**2** $P = 2l + 2w$ [$w$]

**3** $D = \frac{S}{T}$ [$T$]

**4** $p = \frac{q-r}{t}$ [$t$]

**5** $u = at - \frac{1}{2}t$ [$t$]

**6** $V = ax + 4x$ [$x$]

**7** $\frac{y-7x}{2} = \frac{7-2y}{3}$ [$y$]

**8** $x = \frac{2a-1}{3-a}$ [$a$]

**9** $x = \frac{b-c}{d}$ [$d$]

**10** $h = \frac{7g-9}{2+g}$ [$g$]

**11** $e(9 + x) = 2e + 1$ [$e$]

**12** $y = \frac{2x+3}{4-x}$ [$x$]

**13** Make $r$ the subject of the following formulae.

**a** $A = \pi r^2$ **b** $V = \frac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \frac{2}{3}\pi r^2 h$

**14** Make $x$ the subject of the following formulae.

**a** $\frac{xy}{z} = \frac{ab}{cd}$ **b** $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

**15** Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

**16** Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

## Extend

**17** Make $x$ the subject of the following equations.

**a** $\frac{p}{q}(sx + t) = x - 1$ **b** $\frac{p}{q}(ax + 2y) = \frac{3p}{q^2}(x - y)$

# Factorising expressions

## Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is $b$ and whose product is $ac$.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

## Examples

**Example 1** Factorise $15x^2y^3 + 9x^4y$

| | |
|---|---|
| $15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$ | The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets |

**Example 2** Factorise $4x^2 - 25y^2$

| | |
|---|---|
| $4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$ | This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$ |

**Example 3** Factorise $x^2 + 3x - 10$

| | |
|---|---|
| $b = 3$, $ac = -10$ | **1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2) |
| So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$ | **2** Rewrite the $b$ term ($3x$) using these two factors |
| $= x(x + 5) - 2(x + 5)$ | **3** Factorise the first two terms and the last two terms |
| $= (x + 5)(x - 2)$ | **4** $(x + 5)$ is a factor of both terms |

**Example 4** Factorise $6x^2 - 11x - 10$

| | |
|---|---|
| $b = -11$, $ac = -60$ | **1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (−15 and 4) |
| So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ | **2** Rewrite the $b$ term ($-11x$) using these two factors |
| $= 3x(2x - 5) + 2(2x - 5)$ | **3** Factorise the first two terms and the last two terms |
| $= (2x - 5)(3x + 2)$ | **4** $(2x - 5)$ is a factor of both terms |

**Example 5** Simplify $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

| | |
|---|---|
| $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ | **1** Factorise the numerator and the denominator |
| For the numerator: $b = -4$, $ac = -21$ | **2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (−7 and 3) |
| So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ | **3** Rewrite the $b$ term ($-4x$) using these two factors |
| $= x(x - 7) + 3(x - 7)$ | **4** Factorise the first two terms and the last two terms |
| $= (x - 7)(x + 3)$ | **5** $(x - 7)$ is a factor of both terms |
| For the denominator: $b = 9$, $ac = 18$ | **6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3) |
| So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ | **7** Rewrite the $b$ term ($9x$) using these two factors |
| $= 2x(x + 3) + 3(x + 3)$ | **8** Factorise the first two terms and the last two terms |
| $= (x + 3)(2x + 3)$ | **9** $(x + 3)$ is a factor of both terms |
| So $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \dfrac{(x - 7)(x + 3)}{(x + 3)(2x + 3)} = \dfrac{x - 7}{2x + 3}$ | **10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1 |

## Practice

**1** Factorise.

**a** $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

**c** $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

> **Hint**
>
> Take the highest common factor outside the bracket.

**2** Factorise

**a** $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$

**c** $x^2 - 11x + 30$ **d** $x^2 - 5x - 24$

**e** $x^2 - 7x - 18$ **f** $x^2 + x - 20$

**g** $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

**3** Factorise

**a** $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$

**c** $18a^2 - 200b^2c^2$

**4** Factorise

**a** $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

**c** $2x^2 + 7x + 3$ **d** $9x^2 - 15x + 4$

**e** $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

**5** Simplify the algebraic fractions.

**a** $\dfrac{2x^2+4x}{x^2-x}$ **b** $\dfrac{x^2+3x}{x^2+2x-3}$

**c** $\dfrac{x^2-2x-8}{x^2-4x}$ **d** $\dfrac{x^2-5x}{x^2-25}$

**e** $\dfrac{x^2-x-12}{x^2-4x}$ **f** $\dfrac{2x^2+14x}{2x^2+4x-70}$

**6** Simplify

**a** $\dfrac{9x^2-16}{3x^2+17x-28}$ **b** $\dfrac{2x^2-7x-15}{3x^2-17x+10}$

**c** $\dfrac{4-25x^2}{10x^2-11x-6}$ **d** $\dfrac{6x^2-x-1}{2x^2+7x-4}$

## Extend

**7** Simplify $\sqrt{x^2+10x+25}$

**8** Simplify $\dfrac{(x+2)^2+3(x+2)^2}{x^2-4}$

# Solving quadratic equations by factorisation

## Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is $b$ and whose products is $ac$.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

## Examples

**Example 1** Solve $5x^2 = 15x$

| | |
|---|---|
| $5x^2 = 15x$<br><br>$5x^2 - 15x = 0$ | **1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by $x$ as this would lose the solution $x = 0$. |
| $5x(x - 3) = 0$ | **2** Factorise the quadratic equation. $5x$ is a common factor. |
| So $5x = 0$ or $(x - 3) = 0$ | **3** When two values multiply to make zero, at least one of the values must be zero. |
| Therefore $x = 0$ or $x = 3$ | **4** Solve these two equations. |

**Example 2** Solve $x^2 + 7x + 12 = 0$

| | |
|---|---|
| $x^2 + 7x + 12 = 0$<br><br>$b = 7$, $ac = 12$ | **1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3) |
| $x^2 + 4x + 3x + 12 = 0$ | **2** Rewrite the $b$ term ($7x$) using these two factors. |
| $x(x + 4) + 3(x + 4) = 0$ | **3** Factorise the first two terms and the last two terms. |
| $(x + 4)(x + 3) = 0$ | **4** $(x + 4)$ is a factor of both terms. |
| So $(x + 4) = 0$ or $(x + 3) = 0$ | **5** When two values multiply to make zero, at least one of the values must be zero. |
| Therefore $x = -4$ or $x = -3$ | **6** Solve these two equations. |

**Example 3** Solve $9x^2 - 16 = 0$

| | |
|---|---|
| $9x^2 - 16 = 0$<br>$(3x + 4)(3x - 4) = 0$ | **1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. |
| So $(3x + 4) = 0$ or $(3x - 4) = 0$ | **2** When two values multiply to make zero, at least one of the values must be zero. |
| $x = -\frac{4}{3}$ or $x = \frac{4}{3}$ | **3** Solve these two equations. |

**Example 4** Solve $2x^2 - 5x - 12 = 0$

| | |
|---|---|
| $b = -5$, $ac = -24$ | **1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. ($-8$ and 3) |
| So $2x^2 - 8x + 3x - 12 = 0$ | **2** Rewrite the $b$ term ($-5x$) using these two factors. |
| $2x(x - 4) + 3(x - 4) = 0$ | **3** Factorise the first two terms and the last two terms. |
| $(x - 4)(2x + 3) = 0$ | **4** $(x - 4)$ is a factor of both terms. |
| So $(x - 4) = 0$ or $(2x + 3) = 0$ | **5** When two values multiply to make zero, at least one of the values must be zero. |
| $x = 4$ or $x = -\frac{3}{2}$ | **6** Solve these two equations. |

## Practice

**1** Solve

**a** $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

**c** $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

**e** $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

**g** $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

**i** $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

**k** $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

**2** Solve

**a** $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

**c** $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

**e** $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

**g** $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

**Hint**

Get all terms onto one side of the

# Solving quadratic equations by using the formula

## Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for $a$, $b$ and $c$.

## Examples

**Example 7** Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

| | |
|---|---|
| $a = 1$, $b = 6$, $c = 4$<br>$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | **1** Identify $a$, $b$ and $c$ and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it. |
| $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$ | **2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula. |
| $x = \frac{-6 \pm \sqrt{20}}{2}$ | **3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2. |
| $x = \frac{-6 \pm 2\sqrt{5}}{2}$ | **4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$ |
| $x = -3 \pm \sqrt{5}$ | **5** Simplify by dividing numerator and denominator by 2. |
| So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$ | **6** Write down both the solutions. |

**Example 8** Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

| | |
|---|---|
| $a = 3$, $b = -7$, $c = -2$<br>$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | **1** Identify $a$, $b$ and $c$, making sure you get the signs right and write down the formula.<br>Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it. |
| $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$ | **2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula. |
| $x = \dfrac{7 \pm \sqrt{73}}{6}$ | **3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2. |
| So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$ | **4** Write down both the solutions. |

## Practice

**5** Solve, giving your solutions in surd form.

**a** $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

**6** Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\dfrac{a \pm \sqrt{b}}{c}$, where $a$, $b$ and $c$ are integers.

**7** Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

> **Hint**
>
> Get all terms onto one side of the equation.

## Extend

**8** Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

**a** $4x(x - 1) = 3x - 2$

**b** $10 = (x + 1)^2$

**c** $x(3x - 1) = 10$

# Solving linear simultaneous equations using the elimination method

## Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

## Examples

**Example 1** Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

| | |
|---|---|
| $3x + y = 5$<br>$-\quad x + y = 1$<br>$2x \quad = 4$<br>So $x = 2$ | **1** Subtract the second equation from the first equation to eliminate the $y$ term. |
| Using $x + y = 1$<br>$2 + y = 1$<br>So $y = -1$ | **2** To find the value of $y$, substitute $x = 2$ into one of the original equations. |
| Check:<br>equation 1: $3 \times 2 + (-1) = 5$ YES<br>equation 2: $2 + (-1) = 1$ YES | **3** Substitute the values of $x$ and $y$ into both equations to check your answers. |

**Example 2** Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

| | |
|---|---|
| $x + 2y = 13$<br>$+\quad 5x - 2y = 5$<br>$6x \quad = 18$<br>So $x = 3$ | **1** Add the two equations together to eliminate the $y$ term. |
| Using $x + 2y = 13$<br>$3 + 2y = 13$<br>So $y = 5$ | **2** To find the value of $y$, substitute $x = 3$ into one of the original equations. |
| Check:<br>equation 1: $3 + 2 \times 5 = 13$ YES<br>equation 2: $5 \times 3 - 2 \times 5 = 5$ YES | **3** Substitute the values of $x$ and $y$ into both equations to check your answers. |

**Example 3** Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

| | |
|---|---|
| $(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$<br>$(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$<br>$7x = 28$<br><br>So $x = 4$ | **1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of $y$ the same for both equations. Then subtract the first equation from the second equation to eliminate the $y$ term. |
| Using $2x + 3y = 2$<br>$2 \times 4 + 3y = 2$<br>So $y = -2$ | **2** To find the value of $y$, substitute $x = 4$ into one of the original equations. |
| Check:<br>equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES<br>equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES | **3** Substitute the values of $x$ and $y$ into both equations to check your answers. |

## Practice

Solve these simultaneous equations.

**1** $4x + y = 8$
$x + y = 5$

**2** $3x + y = 7$
$3x + 2y = 5$

**3** $4x + y = 3$
$3x - y = 11$

**4** $3x + 4y = 7$
$x - 4y = 5$

**5** $2x + y = 11$
$x - 3y = 9$

**6** $2x + 3y = 11$
$3x + 2y = 4$

# Solving linear simultaneous equations using the substitution method

## Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

## Examples

**Example 4** Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

| | |
|---|---|
| $5x + 3(2x + 1) = 14$ | **1** Substitute $2x + 1$ for $y$ into the second equation. |
| $5x + 6x + 3 = 14$<br>$11x + 3 = 14$ | **2** Expand the brackets and simplify. |
| $11x = 11$<br>So $x = 1$ | **3** Work out the value of $x$. |
| Using $y = 2x + 1$<br>$y = 2 \times 1 + 1$<br>So $y = 3$ | **4** To find the value of $y$, substitute $x = 1$ into one of the original equations. |
| Check:<br>equation 1: $3 = 2 \times 1 + 1$ YES<br>equation 2: $5 \times 1 + 3 \times 3 = 14$ YES | **5** Substitute the values of $x$ and $y$ into both equations to check your answers. |

**Example 5** Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

| | |
|---|---|
| $y = 2x - 16$ | **1** Rearrange the first equation. |
| $4x + 3(2x - 16) = -3$ | **2** Substitute $2x - 16$ for $y$ into the second equation. |
| $4x + 6x - 48 = -3$<br>$10x - 48 = -3$ | **3** Expand the brackets and simplify. |
| $10x = 45$<br>So $x = 4\frac{1}{2}$ | **4** Work out the value of $x$. |
| Using $y = 2x - 16$<br>$y = 2 \times 4\frac{1}{2} - 16$<br>So $y = -7$ | **5** To find the value of $y$, substitute $x = 4\frac{1}{2}$ into one of the original equations. |
| Check:<br>equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES<br>equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES | **6** Substitute the values of $x$ and $y$ into both equations to check your answers. |

## Practice

Solve these simultaneous equations.

**7** $y = x - 4$
$2x + 5y = 43$

**8** $y = 2x - 3$
$5x - 3y = 11$

**9** $2y = 4x + 5$
$9x + 5y = 22$

**10** $2x = y - 2$
$8x - 5y = -11$

**11** $3x + 4y = 8$
$2x - y = -13$

**12** $3y = 4x - 7$
$2y = 3x - 4$

**13** $3x = y - 1$
$2y - 2x = 3$

**14** $3x + 2y + 1 = 0$
$4y = 8 - x$

## Extend

**15** Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

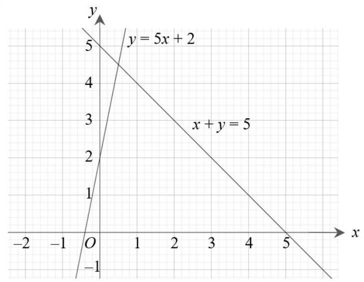
# Solving simultaneous equations graphically

## Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

## Examples

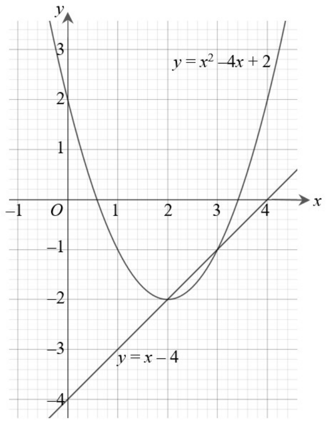
**Example 1** Solve the simultaneous equations $y = 5x + 2$ and $x + y = 5$ graphically.

| | |
|---|---|
| $y = 5 - x$ | **1** Rearrange the equation $x + y = 5$ to make $y$ the subject. |
| $y = 5 - x$ has gradient –1 and $y$-intercept 5.<br>$y = 5x + 2$ has gradient 5 and $y$-intercept 2.<br> | **2** Plot both graphs on the same grid using the gradients and $y$-intercepts. |
| Lines intersect at<br>$x = 0.5$, $y = 4.5$ | **3** The solutions of the simultaneous equations are the point of intersection. |
| Check:<br>First equation $y = 5x + 2$:<br>$4.5 = 5 \times 0.5 + 2$ YES<br>Second equation $x + y = 5$:<br>$0.5 + 4.5 = 5$ YES | **4** Check your solutions by substituting the values into both equations. |

**Example 2** Solve the simultaneous equations $y = x - 4$ and $y = x^2 - 4x + 2$ graphically.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 2 | –1 | –2 | –1 | 2 |

**1** Construct a table of values and calculate the points for the quadratic equation.




**2** Plot the graph.

**3** Plot the linear graph on the same grid using the gradient and $y$-intercept.
$y = x - 4$ has gradient 1 and $y$-intercept –4.

The line and curve intersect at
$x = 3, y = -1$ and $x = 2, y = -2$

**4** The solutions of the simultaneous equations are the points of intersection.

Check:
First equation $y = x - 4$:
$-1 = 3 - 4$ YES
$-2 = 2 - 4$ YES
Second equation $y = x^2 - 4x + 2$:
$-1 = 3^2 - 4 \times 3 + 2$ YES
$-2 = 2^2 - 4 \times 2 + 2$ YES

**5** Check your solutions by substituting the values into both equations.

# Practice

**1** Solve these pairs of simultaneous equations graphically.
**a** $y = 3x - 1$ and $y = x + 3$
**b** $y = x - 5$ and $y = 7 - 5x$
**c** $y = 3x + 4$ and $y = 2 - x$

**2** Solve these pairs of simultaneous equations graphically.
**a** $x + y = 0$ and $y = 2x + 6$
**b** $4x + 2y = 3$ and $y = 3x - 1$
**c** $2x + y + 4 = 0$ and $2y = 3x - 1$

**Hint**
Rearrange the equation to make *y* the

**3** Solve these pairs of simultaneous equations graphically.
**a** $y = x - 1$ and $y = x^2 - 4x + 3$
**b** $y = 1 - 3x$ and $y = x^2 - 3x - 3$
**c** $y = 3 - x$ and $y = x^2 + 2x + 5$

**4** Solve the simultaneous equations $x + y = 1$ and $x^2 + y^2 = 25$ graphically.

# Extend

**5** **a** Solve the simultaneous equations $2x + y = 3$ and $x^2 + y = 4$
**i** graphically
**ii** algebraically to 2 decimal places.
**b** Which method gives the more accurate solutions? Explain your answer.

# Rules of indices

## Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the $n$th root of $a$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

## Examples

**Example 1** Evaluate $10^0$

| | |
|---|---|
| $10^0 = 1$ | Any value raised to the power of zero is equal to 1 |

**Example 2** Evaluate $9^{\frac{1}{2}}$

| | |
|---|---|
| $9^{\frac{1}{2}} = \sqrt{9}$<br>$= 3$ | Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ |

**Example 3** Evaluate $27^{\frac{2}{3}}$

| | |
|---|---|
| $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$<br>$= 3^2$<br>$= 9$ | **1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$<br>**2** Use $\sqrt[3]{27} = 3$ |

**Example 4** Evaluate $4^{-2}$

| | |
|---|---|
| $4^{-2} = \dfrac{1}{4^2}$<br>$= \dfrac{1}{16}$ | **1** Use the rule $a^{-m} = \dfrac{1}{a^m}$<br>**2** Use $4^2 = 16$ |

**Example 5** Simplify $\dfrac{6x^5}{2x^2}$

| | |
|---|---|
| $\dfrac{6x^5}{2x^2} = 3x^3$ | $6 \div 2 = 3$ and use the rule $\dfrac{a^m}{a^n} = a^{m-n}$ to give $\dfrac{x^5}{x^2} = x^{5-2} = x^3$ |

**Example 6** Simplify $\dfrac{x^3 \times x^5}{x^4}$

| | |
|---|---|
| $\dfrac{x^3 \times x^5}{x^4} = \dfrac{x^{3+5}}{x^4} = \dfrac{x^8}{x^4}$<br>$= x^{8-4} = x^4$ | **1** Use the rule $a^m \times a^n = a^{m+n}$<br>**2** Use the rule $\dfrac{a^m}{a^n} = a^{m-n}$ |

**Example 7** Write $\dfrac{1}{3x}$ as a single power of $x$

| | |
|---|---|
| $\dfrac{1}{3x} = \dfrac{1}{3}x^{-1}$ | Use the rule $\dfrac{1}{a^m} = a^{-m}$, note that the fraction $\dfrac{1}{3}$ remains unchanged |

**Example 8** Write $\dfrac{4}{\sqrt{x}}$ as a single power of $x$

| | |
|---|---|
| $\dfrac{4}{\sqrt{x}} = \dfrac{4}{x^{\frac{1}{2}}}$<br>$= 4x^{-\frac{1}{2}}$ | **1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$<br>**2** Use the rule $\dfrac{1}{a^m} = a^{-m}$ |

## Practice

**1** Evaluate.

**a** $14^0$ **b** $3^0$ **c** $5^0$ **d** $x^0$

**2** Evaluate.

**a** $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

**3** Evaluate.

**a** $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

**4** Evaluate.

**a** $5^{-2}$ **b** $4^{-3}$ **c** $2^{-5}$ **d** $6^{-2}$

**5** Simplify.

**a** $\frac{3x^2 \times x^3}{2x^2}$ **b** $\frac{10x^5}{2x^2 \times x}$

**c** $\frac{3x \times 2x^3}{2x^3}$ **d** $\frac{7x^3y^2}{14x^5y}$

**e** $\frac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

**g** $\frac{(2x^2)^3}{4x^0}$ **h** $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

**Watch out!**

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

**6** Evaluate.

**a** $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

**d** $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

**7** Write the following as a single power of $x$.

**a** $\frac{1}{x}$ **b** $\frac{1}{x^7}$ **c** $\sqrt[4]{x}$

**d** $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt[3]{x^2}}$

**8** Write the following without negative or fractional powers.

**a** $x^{-3}$ **b** $x^0$ **c** $x^{\frac{1}{5}}$

**d** $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

**9** Write the following in the form $ax^n$.

**a** $5\sqrt{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{1}{3x^4}$

**d** $\frac{2}{\sqrt{x}}$ **e** $\frac{4}{\sqrt[3]{x}}$ **f** $3$

## Extend

**10** Write as sums of powers of $x$.

**a** $\frac{x^5+1}{x^2}$ **b** $x^2\left(x+\frac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\frac{1}{x^3}\right)$

# Surds and rationalising the denominator

## Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

## Examples

**Example 1** Simplify $\sqrt{50}$

| | |
|---|---|
| $\sqrt{50} = \sqrt{25 \times 2}$ | **1** Choose two numbers that are factors of 50. One of the factors must be a square number |
| $= \sqrt{25} \times \sqrt{2}$ | **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ |
| $= 5 \times \sqrt{2}$ | **3** Use $\sqrt{25} = 5$ |
| $= 5\sqrt{2}$ | |

**Example 2** Simplify $\sqrt{147} - 2\sqrt{12}$

| | |
|---|---|
| $\sqrt{147} - 2\sqrt{12}$<br>$= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$ | **1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number |
| $= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$ | **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ |
| $= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$ | **3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$ |
| $= 7\sqrt{3} - 4\sqrt{3}$<br>$= 3\sqrt{3}$ | **4** Collect like terms |

**Example 3** Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

| | |
|---|---|
| $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$<br>$= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$ | **1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$ |
| $= 7 - 2$<br>$= 5$ | **2** Collect like terms:<br>$-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$<br>$= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$ |

**Example 4** Rationalise $\frac{1}{\sqrt{3}}$

| | |
|---|---|
| $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ | **1** Multiply the numerator and denominator by $\sqrt{3}$ |
| $= \frac{1 \times \sqrt{3}}{\sqrt{9}}$ | **2** Use $\sqrt{9} = 3$ |
| $= \frac{\sqrt{3}}{3}$ | |

**Example 5** Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

| | |
|---|---|
| $\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$ | **1** Multiply the numerator and denominator by $\sqrt{12}$ |
| $= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$ | **2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number |
| $= \frac{2\sqrt{2}\sqrt{3}}{12}$ | **3** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$<br>**4** Use $\sqrt{4} = 2$ |
| $= \frac{\sqrt{2}\sqrt{3}}{6}$ | **5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$ |

**Example 6** Rationalise and simplify $\dfrac{3}{2+\sqrt{5}}$

| | |
|---|---|
| $\dfrac{3}{2+\sqrt{5}} = \dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$ | **1** Multiply the numerator and denominator by $2-\sqrt{5}$ |
| $= \dfrac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$ | **2** Expand the brackets |
| $= \dfrac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ | **3** Simplify the fraction |
| $= \dfrac{6-3\sqrt{5}}{-1}$<br>$= 3\sqrt{5}-6$ | **4** Divide the numerator by −1<br>Remember to change the sign of all terms when dividing by −1 |

## Practice

**1** Simplify.

**a** $\sqrt{45}$ **b** $\sqrt{125}$

**c** $\sqrt{48}$ **d** $\sqrt{175}$

**e** $\sqrt{300}$ **f** $\sqrt{28}$

**g** $\sqrt{72}$ **h** $\sqrt{162}$

> **Hint**
>
> One of the two numbers you choose at the start must be a square number.

**2** Simplify.

**a** $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

**c** $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

**e** $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

> **Watch out!**
>
> Check you have chosen the highest square number at the

**3** Expand and simplify.

**a** $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

**c** $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

**4** Rationalise and simplify, if possible.

**a** $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{1}{\sqrt{11}}$

**c** $\dfrac{2}{\sqrt{7}}$ **d** $\dfrac{2}{\sqrt{8}}$

**e** $\dfrac{2}{\sqrt{2}}$ **f** $\dfrac{5}{\sqrt{5}}$

**g** $\dfrac{\sqrt{8}}{\sqrt{24}}$ **h** $\dfrac{\sqrt{5}}{\sqrt{45}}$

**5** Rationalise and simplify.

**a** $\dfrac{1}{3-\sqrt{5}}$ **b** $\dfrac{2}{4+\sqrt{3}}$ **c** $\dfrac{6}{5-\sqrt{2}}$

## Extend

**6** Expand and simplify $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$

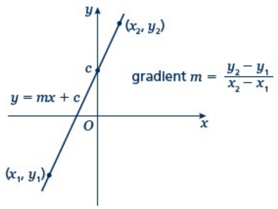
**7** Rationalise and simplify, if possible.

**a** $\dfrac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\dfrac{1}{\sqrt{x}-\sqrt{y}}$

# Straight line graphs

## Key points

- A straight line has the equation $y = mx + c$, where $m$ is the gradient and $c$ is the $y$-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.
- When given the coordinates $(x_1, y_1)$ and $(x_2, y_2)$ of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$




## Examples

**Example 1** A straight line has gradient $-\frac{1}{2}$ and $y$-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

| | |
|---|---|
| $m = -\frac{1}{2}$ and $c = 3$ | **1** A straight line has equation $y = mx + c$. Substitute the gradient and $y$-intercept given in the question into this equation. |
| So $y = -\frac{1}{2}x + 3$ | |
| $\frac{1}{2}x + y - 3 = 0$ | **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. |
| $x + 2y - 6 = 0$ | **3** Multiply both sides by 2 to eliminate the denominator. |

**Example 2** Find the gradient and the $y$-intercept of the line with the equation $3y - 2x + 4 = 0$.

| | |
|---|---|
| $3y - 2x + 4 = 0$<br>$3y = 2x - 4$ | **1** Make $y$ the subject of the equation. |
| $y = \frac{2}{3}x - \frac{4}{3}$ | **2** Divide all the terms by three to get the equation in the form $y = \ldots$ |
| Gradient = $m = \frac{2}{3}$ | **3** In the form $y = mx + c$, the gradient is $m$ and the $y$-intercept is $c$. |
| $y$-intercept = $c = -\frac{4}{3}$ | |

**Example 3** Find the equation of the line which passes through the point (5, 13) and has gradient 3.

| | |
|---|---|
| $m = 3$<br>$y = 3x + c$ | **1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. |
| $13 = 3 \times 5 + c$ | **2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation. |
| $13 = 15 + c$<br>$c = -2$ | **3** Simplify and solve the equation. |
| $y = 3x - 2$ | **4** Substitute $c = -2$ into the equation $y = 3x + c$ |

**Example 4** Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

| | |
|---|---|
| $x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$<br>$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$ | **1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. |
| $y = \frac{1}{2}x + c$ | **2** Substitute the gradient into the equation of a straight line $y = mx + c$. |
| $4 = \frac{1}{2} \times 2 + c$ | **3** Substitute the coordinates of either point into the equation. |
| $c = 3$ | **4** Simplify and solve the equation. |
| $y = \frac{1}{2}x + 3$ | **5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$ |

## Practice

**1** Find the gradient and the $y$-intercept of the following equations.

**a** $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

**c** $2y = 4x - 3$ **d** $x + y = 5$

**e** $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations to the form $y = mx + c$

**2** Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

| Gradient | $y$-intercept | Equation of the line |
|---|---|---|
| 5 | 0 | |
| –3 | 2 | |
| 4 | –7 | |

**3** Find, in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers, an equation for each of the lines with the following gradients and $y$-intercepts.

**a** gradient $-\frac{1}{2}$, $y$-intercept $-7$ **b** gradient 2, $y$-intercept 0

**c** gradient $\frac{2}{3}$, $y$-intercept 4 **d** gradient $-1.2$, $y$-intercept $-2$

**4** Write an equation for the line which passes though the point (2, 5) and has gradient 4.

**5** Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

**6** Write an equation for the line passing through each of the following pairs of points.

**a** (4, 5), (10, 17) **b** (0, 6), (–4, 8)

**c** (–1, –7), (5, 23) **d** (3, 10), (4, 7)

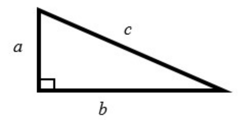
## Extend

**7** The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

# Pythagoras' theorem

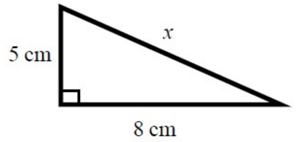
## Key points

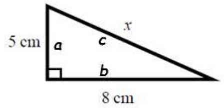
- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
  $c^2 = a^2 + b^2$



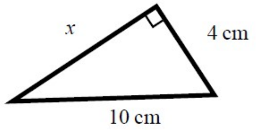

## Examples

**Example 1** Calculate the length of the hypotenuse.
Give your answer to 3 significant figures.




| | |
|---|---|
| $c^2 = a^2 + b^2$  | **1** Always start by stating the formula for Pythagoras' theorem and labelling the hypotenuse $c$ and the other two sides $a$ and $b$. |
| $x^2 = 5^2 + 8^2$<br>$x^2 = 25 + 64$<br>$x^2 = 89$<br>$x = \sqrt{89}$ | **2** Substitute the values of $a$, $b$ and $c$ into the formula for Pythagoras' theorem.<br>**3** Use a calculator to find the square root. |
| $x = 9.433\ 981\ 13...$<br>$x = 9.43$ cm | **4** Round your answer to 3 significant figures and write the units with your answer. |

**Example 2** Calculate the length $x$.
Give your answer in surd form.

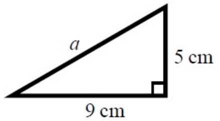



| | |
|---|---|
| $c^2 = a^2 + b^2$<br><br>$10^2 = x^2 + 4^2$<br>$100 = x^2 + 16$<br>$x^2 = 84$<br>$x = \sqrt{84}$<br>$x = 2\sqrt{21}$ cm | **1** Always start by stating the formula for Pythagoras' theorem.<br>**2** Substitute the values of $a$, $b$ and $c$ into the formula for Pythagoras' theorem.<br><br>**3** Simplify the surd where possible and write the units in your answer. |

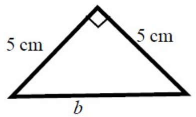
## Practice

**1** Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

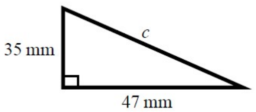
**a**



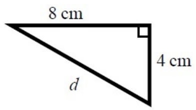

**b**




**c**

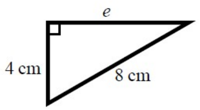



**d**

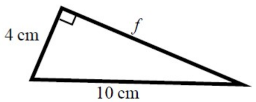



**2** Work out the length of the unknown side in each triangle.
Give your answers in surd form.

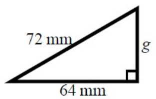
**a**



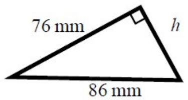

**b**




**c**

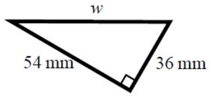



**d**

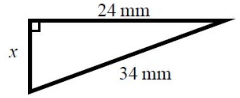



**3** Work out the length of the unknown side in each triangle.
Give your answers in surd form.

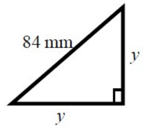
**a**



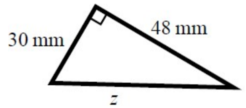

**b**




**c**




**d**




**4** A rectangle has length 84 mm and width 45 mm.
Calculate the length of the diagonal of the rectangle.
Give your answer correct to 3 significant figures.

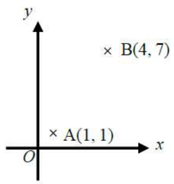
> **Hint**
>
> Draw a sketch of the rectangle.

## Extend

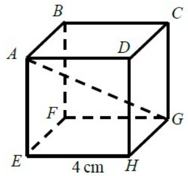
**5** A yacht is 40 km due North of a lighthouse.
A rescue boat is 50 km due East of the same lighthouse.
Work out the distance between the yacht and the rescue boat.
Give your answer correct to 3 significant figures.

> **Hint**
>
> Draw a diagram using the information given in the question.

**6** Points A and B are shown on the diagram.
Work out the length of the line AB.
Give your answer in surd form.



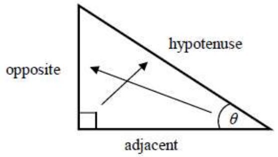

**7** A cube has length 4 cm.
Work out the length of the diagonal $AG$.
Give your answer in surd form.



# Trigonometry in right-angled triangles

## Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle $\theta$ is called the opposite
  - the side next to the angle $\theta$ is called the adjacent.

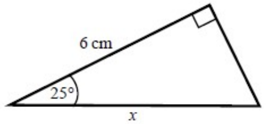


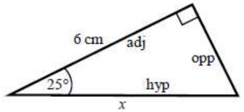

- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle $\theta$, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle $\theta$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle $\theta$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.
- The sine, cosine and tangent of some angles may be written exactly.

| | 0 | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| **sin** | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| **cos** | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| **tan** | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | |

## Examples

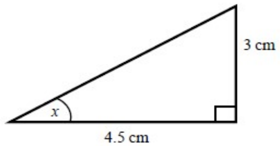
**Example 1** Calculate the length of side $x$.
Give your answer correct to 3 significant figures.

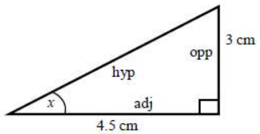



| | |
|---|---|
| | **1** Always start by labelling the sides. |
| $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ | **2** You are given the adjacent and the hypotenuse so use the cosine ratio. |
| $\cos 25° = \frac{6}{x}$ | **3** Substitute the sides and angle into the cosine ratio. |
| $x = \frac{6}{\cos 25°}$ | **4** Rearrange to make $x$ the subject. |
| $x = 6.620\,267\,5...$ | **5** Use your calculator to work out $6 \div \cos 25°$. |
| $x = 6.62$ cm | **6** Round your answer to 3 significant figures and write the units in your answer. |

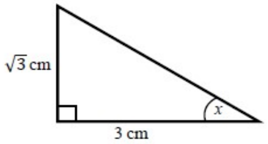
**Example 2** Calculate the size of angle $x$.
Give your answer correct to 3 significant figures.

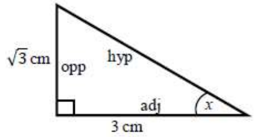



| | |
|---|---|
| | **1** Always start by labelling the sides. |
| $\tan\theta = \frac{\text{opp}}{\text{adj}}$ | **2** You are given the opposite and the adjacent so use the tangent ratio. |
| $\tan x = \frac{3}{4.5}$ | **3** Substitute the sides and angle into the tangent ratio. |
| $x = \tan^{-1}\left(\frac{3}{4.5}\right)$ | **4** Use $\tan^{-1}$ to find the angle. |
| $x = 33.690\,067\,5...$ | **5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$. |
| $x = 33.7°$ | **6** Round your answer to 3 significant figures and write the units in your answer. |

**Example 3** Calculate the exact size of angle $x$.

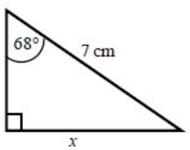



| | |
|---|---|
|   | **1** Always start by labelling the sides. |
| $\tan\theta = \frac{\text{opp}}{\text{adj}}$ | **2** You are given the opposite and the adjacent so use the tangent ratio. |
| $\tan x = \frac{\sqrt{3}}{3}$ | **3** Substitute the sides and angle into the tangent ratio. |
| $x = 30°$ | **4** Use the table from the key points to find the angle. |

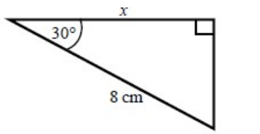
## Practice

**1** Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

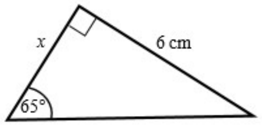
**a**



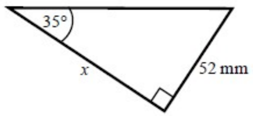

**b**



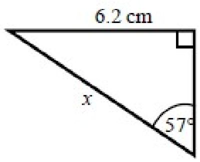

**c**



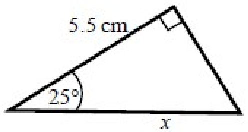

**d**




**e**

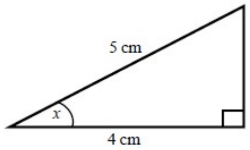



**f**

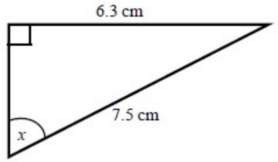



**2** Calculate the size of angle $x$ in each triangle.
Give your answers correct to 1 decimal place.

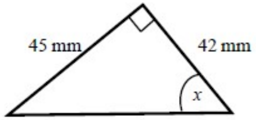
**a**



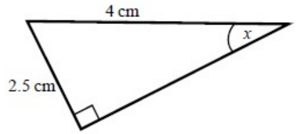

**b**




**c**

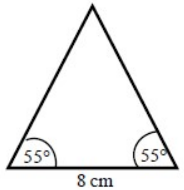



**d**



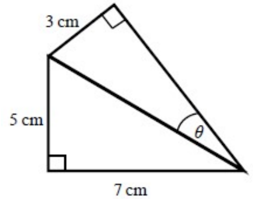

**3** Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

> **Hint:**
>
> Split the triangle into two right-angled triangles.



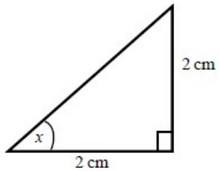

**4** Calculate the size of angle $\theta$.
Give your answer correct to 1 decimal place.

> **Hint:**
>
> First work out the length of the common side to both triangles, leaving your answer in surd form.

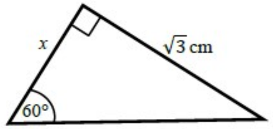



**5** Find the exact value of $x$ in each triangle.

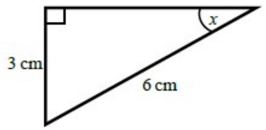
**a**



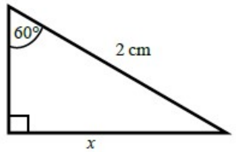

**b**




**c**




**d**



# Answers

## Rearranging equations

**1** $d = \frac{C}{\pi}$ **2** $w = \frac{P - 2l}{2}$ **3** $T = \frac{S}{D}$

**4** $t = \frac{q - r}{p}$ **5** $t = \frac{2u}{2a - 1}$ **6** $x = \frac{V}{a + 4}$

**7** $y = 2 + 3x$ **8** $a = \frac{3x + 1}{x + 2}$ **9** $d = \frac{b - c}{x}$

**10** $g = \frac{2h + 9}{7 - h}$ **11** $e = \frac{1}{x + 7}$ **12** $x = \frac{4y - 3}{2 + y}$

**13** **a** $r = \sqrt{\frac{A}{\pi}}$ **b** $r = \sqrt[3]{\frac{3V}{4\pi}}$

**c** $r = \frac{P}{\pi + 2}$ **d** $r = \sqrt{\frac{3V}{2\pi h}}$

**14** **a** $x = \frac{abz}{cdy}$ **b** $x = \frac{3dz}{4\pi cpy^2}$

**15** $\sin B = \frac{b \sin A}{a}$

**16** $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

**17** **a** $x = \frac{q + pt}{q - ps}$ **b** $x = \frac{3py + 2pqy}{3p - apq} = \frac{y(3 + 2q)}{3 - aq}$

## Factorising expressions

**1** **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$
**c** $5x^2y^2(5 - 2x + 3y)$

**2** **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$
**c** $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$
**e** $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$
**g** $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$

**3** **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$
**c** $2(3a - 10bc)(3a + 10bc)$

**4** **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$
**c** $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$
**e** $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$

**5** **a** $\frac{2(x + 2)}{x - 1}$ **b** $\frac{x}{x - 1}$
**c** $\frac{x + 2}{x}$ **d** $\frac{x}{x + 5}$
**e** $\frac{x + 3}{x}$ **f** $\frac{x}{x - 5}$

**6** **a** $\frac{3x + 4}{x + 7}$ **b** $\frac{2x + 3}{3x - 2}$
**c** $\frac{2 - 5x}{2x - 3}$ **d** $\frac{3x + 1}{x + 4}$

**7** $(x + 5)$

**8** $\frac{4(x + 2)}{x - 2}$

## Solving quadratic equations

**1** **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$
**c** $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$
**e** $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$
**g** $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$
**i** $x = -7$ or $x = 4$ **j** $x = 3$
**k** $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

**2** **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$
**c** $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$
**e** $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$
**g** $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

**5** **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

**6** $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

**7** $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

**8** **a** $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

**b** $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

**c** $x = -1\frac{2}{3}$ or $x = 2$

## Solving linear simultaneous equations

**1** $x = 1, y = 4$

**2** $x = 3, y = -2$

**3** $x = 2, y = -5$

**4** $x = 3, y = -\frac{1}{2}$

**5** $x = 6, y = -1$

**6** $x = -2, y = 5$

**7** $x = 9, y = 5$

**8** $x = -2, y = -7$

**9** $x = \frac{1}{2}, y = 3\frac{1}{2}$

**10** $x = \frac{1}{2}, y = 3$

**11** $x = -4, y = 5$

**12** $x = -2, y = -5$

**13** $x = \frac{1}{4}, y = 1\frac{3}{4}$

**14** $x = -2, y = 2\frac{1}{2}$

**15** $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

## Solving simultaneous equations graphically

**1** **a** $x = 2, y = 5$

**b** $x = 2, y = -3$

**c** $x = -0.5, y = 2.5$

**2** **a** $x = -2, y = 2$

**b** $x = 0.5, y = 0.5$

**c** $x = -1, y = -2$

**3** **a** $x = 1, y = 0$ and $x = 4, y = 3$

**b** $x = -2, y = 7$ and $x = 2, y = -5$

**c** $x = -2, y = 5$ and $x = -1, y = 4$

**4** $x = -3, y = 4$ and $x = 4, y = -3$

**5** **a** **i** $x = 2.5, y = -2$ and $x = -0.5, y = 4$

**ii** $x = 2.41, y = -1.83$ and $x = -0.41, y = 3.83$

**b** Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.

## Rules of indices

**1** **a** 1 **b** 1 **c** 1 **d** 1

**2** **a** 7 **b** 4 **c** 5 **d** 2

**3** **a** 125 **b** 32 **c** 343 **d** 8

**4** **a** $\frac{1}{25}$ **b** $\frac{1}{64}$ **c** $\frac{1}{32}$ **d** $\frac{1}{36}$

**5** **a** $\frac{3x^3}{2}$ **b** $5x^2$

**c** $3x$ **d** $\frac{y}{2x^2}$

**e** $y^{\frac{1}{2}}$ **f** $c^{-3}$

**g** $2x^6$ **h** $x$

**6** **a** $\frac{1}{2}$ **b** $\frac{1}{9}$ **c** $\frac{8}{3}$

**d** $\frac{1}{4}$ **e** $\frac{4}{3}$ **f** $\frac{16}{9}$

**7** **a** $x^{-1}$ **b** $x^{-7}$ **c** $x^{\frac{1}{4}}$

**d** $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{3}}$ **f** $x^{-\frac{2}{3}}$

**8** **a** $\frac{1}{x^3}$ **b** $1$ **c** $\sqrt[5]{x}$

**d** $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt{x}}$ **f** $\frac{1}{\sqrt[4]{x^3}}$

**9** **a** $5x^{\frac{1}{2}}$ **b** $2x^{-3}$ **c** $\frac{1}{3}x^{-4}$

**d** $2x^{-\frac{1}{2}}$ **e** $4x^{-\frac{1}{3}}$ **f** $3x^0$

**10** **a** $x^3 + x^{-2}$ **b** $x^3 + x$ **c** $x^{-2} + x^{-7}$

## Surds and rationalising the denominator

**1** **a** $3\sqrt{5}$ **b** $5\sqrt{5}$

**c** $4\sqrt{3}$ **d** $5\sqrt{7}$

**e** $10\sqrt{3}$ **f** $2\sqrt{7}$

**g** $6\sqrt{2}$ **h** $9\sqrt{2}$

**2** **a** $15\sqrt{2}$ **b** $\sqrt{5}$

**c** $3\sqrt{2}$ **d** $\sqrt{3}$

**e** $6\sqrt{7}$ **f** $5\sqrt{3}$

**3** **a** $-1$ **b** $9-\sqrt{3}$

**c** $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

**4** **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$

**c** $\frac{2\sqrt{7}}{7}$ **d** $\frac{\sqrt{2}}{2}$

**e** $\sqrt{2}$ **f** $\sqrt{5}$

**g** $\frac{\sqrt{3}}{3}$ **h** $\frac{1}{3}$

**5** **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

**6** $x-y$

**7** **a** $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

## Straight line graphs

**1** **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

**c** $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

**e** $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

**2**

| Gradient | *y*-intercept | Equation of the line |
|---|---|---|
| 5 | 0 | $y = 5x$ |
| –3 | 2 | $y = -3x + 2$ |
| 4 | –7 | $y = 4x - 7$ |

**3** **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

**c** $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

**4** $y = 4x - 3$

**5** $y = -\frac{2}{3}x + 7$

**6** **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

**c** $y = 5x - 2$ **d** $y = -3x + 19$

**7** $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the $y$-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

## Pythagoras' theorem

**1** **a** 10.3 cm **b** 7.07 cm

**c** 58.6 mm **d** 8.94 cm

**2** **a** $4\sqrt{3}$ cm **b** $2\sqrt{21}$ cm

**c** $8\sqrt{17}$ mm **d** $18\sqrt{5}$ mm

**3** **a** $18\sqrt{13}$ mm **b** $2\sqrt{145}$ mm

**c** $42\sqrt{2}$ mm **d** $6\sqrt{89}$ mm

**4** 95.3 mm

**5** 64.0 km

**6** $3\sqrt{5}$ units

**7** $4\sqrt{3}$ cm

## Trigonometry in right-angled triangles

**1** **a** 6.49 cm **b** 6.93 cm **c** 2.80 cm
**d** 74.3 mm **e** 7.39 cm **f** 6.07 cm

**2** **a** 36.9° **b** 57.1° **c** 47.0° **d** 38.7°

**3** 5.71 cm

**4** 20.4°

**5** **a** 45° **b** 1 cm **c** 30° **d** $\sqrt{3}$ cm